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On D-brane boundary state analysis in pure spinor formalism

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Abstract: We explore a particular approach to study D-brane boundary states in Berkovits' pure spinor formalism of superstring theories. In this approach one constructs the boundary states in the relevant conformal field theory by relaxing the pure spinor constraints. This enables us to write down the open string boundary conditions for non-BPS D-branes in type II string theories, generalizing our previous work in light-cone Green-Schwarz formalism. As a first step to explore how to apply these boundary states for physical computations we prescribe rules for computing disk one point functions for the supergravity modes. We also comment on the force computation between two D-branes and point out that it is hard to make the world-sheet open-closed duality manifest in this computation.

Keywords: D-branes.

Contents

1. Introduction and summary

Quantizing superstrings with manifest super-Poincaré covariance is a long standing problem. Several years ago an extension of Siegel's approach [1] to Green-Schwarz (GS) superstrings [2] was proposed by Berkovits [3 − 7], where a set of bosonic ghost degrees of freedom, which are space-time spinors satisfying a pure spinor constraint, were introduced. Various intuitive rules have been suggested for necessary computations. It has been argued [4] that it gives the correct physical perturbative spectrum of string theory. Rules for computing scattering amplitudes have also been formulated [3, 5, 6] which have produced super-Poincaré covariant results more easily than the standard Nevew-Schwarz-Ramond (NSR) formalism. These striking results make it essential to study this approach deeply in order to get new insights into superstring theories.[1] Certainly an important issue to be considered is the study of D-branes in this context. This has been done from various points of view in [10 − 12] (see also [13]). In particular, in [12] Schiappa and Wyllard studied the boundary state analysis for D-branes and disk scattering amplitudes. Due to the pure spinor constraint the construction of boundary states, which involved relating the variables of pure spinor and NSR formalisms [7], looks complicated. To avoid such complications we shall explore an alternative approach which might provide a convenient computational tool for analysis involving boundary states. We shall discuss this below after going through some basic relevant features of pure spinor formalism.

[1] See [8] for other relevant works and [9] for studies on different non-trivial backgrounds.

In pure spinor formalism one starts out with a given conformal field theory (CFT) which is supposed to be the conformal gauge fixed form of a local action.[2] In addition to the usual bosonic matter of the standard NSR formalism, this CFT also includes a fermionic matter and a bosonic ghost parts both of which contain space-time fermions. All the world-sheet fields are free except that the ghost fields are required to satisfy a covariant pure spinor constraint[3] which removes the desired number of degrees of freedom. Naturally, this constraint makes the Hilbert space structure of the theory more complicated than the one corresponding to the unconstrained CFT. The physical states are given by the states of certain ghost number in the cohomology of a proposed BRST operator [3]. As usual, there exists linearized gauge transformation provided by the BRST exact states. Although it is understood how the massless closed string states arise in this formalism, arbitrarily high massive states are difficult to describe. This is because of the pure spinor constraint and the fact that, to begin with, the vertex operators are arbitrary functions on the $d = 9 + 1$, $\mathcal{N} = 1$ superspace (for open strings). Although the latter may be expected for a manifestly super-Poincaré covariant formalism, it results in a lot of redundant fields [17 − 19] which need to be removed by gauge fixation, a procedure that has to be done separately at every level. In NSR formalism the associated gauge symmetry is fixed by a set of well known simple conditions. Moreover, in this gauge the quadratic part of the space-time action simplifies in a certain manner so that the propagator gets an interpretation of the world-sheet time evolution [20]. This sits at the heart of the fact that world-sheet open-closed duality is manifest in NSR computations. It is not yet clear what should be the analog of this gauge choice in pure spinor formalism.

Let us now discuss D-brane boundary states in this context.[4] Given a closed string Hilbert space this state is found as a solution to the open string boundary condition expressed in the closed string channel. Construction of the Hilbert space covariantly in pure spinor formalism is not very easy as the CFT is actually interacting because of the pure spinor constraint. Nevertheless, since the interaction is introduced only through the constraint, one might expect that this Hilbert space can be embedded in the bigger Hilbert space of the unconstrained theory which is completely free. Our approach will be to construct boundary states in the unconstrained theory and to prescribe rules for computing physical quantities using them. Computationally, this may prove favorable provided all such rules can be consistently set up. This approach makes it easy to write down the open string boundary conditions and boundary states for BPS D-branes. The same for non-BPS D-branes in light-cone GS formalism were recently found in [22]. The open string boundary conditions turned out to relate bi-local operators that are quadratic in space-time fermions. We shall see that it has a simple generalization to the present case.

To explore exactly how these boundary states should be used for physical computations we first consider computing the strength of the closed string sources that D-branes produce. Here we demonstrate that indeed a consistent set of rules can be prescribed at

[2] Attempts have been made in [14] to understand the origin of this approach.

[3] Extensions of the present framework by relaxing this constraint have been considered in [15, 16].

[4] See, for example, [21] for reviews on this subject in NSR formalism.

least for the supergravity modes, leaving its generalization to higher massive modes and to arbitrary disk scattering amplitudes for future work. Another interesting computation to be understood is the so-called *cylinder diagram* which gives the force between two D-branes and demonstrates the world-sheet open-closed duality. Because of the problem of gauge fixation mentioned earlier, this computation is not straightforward in pure spinor formalism. Moreover, in NSR formalism ghosts produce a background independent contribution which cancels two (light-cone) coordinates worth of contribution from the matter part. We emphasize that it is difficult to get such background independent contribution in pure spinor formalism. We demonstrate this with the simplest computation, namely the long range NS-NS force between two parallel D-branes where only the massless NS-NS states are involved. The relevant ghost contribution in NSR formalism comes from the ghost-dilaton which does not have any analog in pure spinor formalism. This does not necessarily imply any inconsistency as the whole computation can be performed in supergravity where one is only required to evaluate a tree-level Feynman diagram between two sources [23]. The only role played by the boundary states in this computation is to provide the correct strength for the sources.[5] Obviously the world-sheet open-closed duality will not be manifest in such a computation. Extending this argument to higher massive levels we suggest that the present boundary state should actually be compared with the NSR boundary states in old covariant quantization. Computation of the R-R amplitude is complicated even in the NSR formalism because of the superghost zero modes [24]. Its study in the pure spinor formalism may require special attention which we leave for future work.

The rest of the paper is organized as follows. We review the basic CFT structure of the pure spinor formalism in sec.2. The BPS and non-BPS boundary conditions and boundary states have been discussed in sec.3. Sec.4 concentrates on physical computations where we demonstrate how to compute disk one point functions for the supergravity modes and comment on the force computation. Sec.5 discusses some future directions. Our convention for the gamma matrices and relevant identities are given in the appendix.

2. The conformal field theory

In pure spinor formalism one begins with a CFT which has the following three parts for type II string theories [3]:

$$S = S_B + S_F + S_G \ , \tag{2.1}$$

where S_B is same as the bosonic matter conformal field theory in the usual NSR formalism and therefore has central charge $c_B = \tilde{c}_B = 10$. The left moving part of S_F is the direct sum of 16 fermionic (b, c) theories, namely $(p_\alpha(z),\ \theta^\alpha(z))$ where $\alpha = 1, 2, \cdots, 16$ is a space-time spinor index that has been explained in appendix A. Conformal dimensions of the fields are as follows: $h_{p_\alpha} = 1,\ h_{\theta_\alpha} = 0$. For type IIB string theory the right moving part is same as the left moving part whereas for type IIA the space-time chirality of the fields are

[5] This statement is true even at the full string theoretic level as long as there is a space-time field theoretic way of computing the force.

just opposite, i.e. $(p^\alpha(\bar z),\ \theta_\alpha(\bar z))$. Therefore the central charges for S_F are $c_F = \tilde c_F = -32$. The bosonic ghost part S_G is the most difficult part. The left moving part of it is given by the direct sum of 16 bosonic (β,γ) systems: $(w_\alpha(z),\lambda^\alpha(z))$ with conformal dimensions $h_{w_\alpha} = 1,\ h_{\lambda_\alpha} = 0$ and with a pure spinor constraint on λ^α. Just like the case of S_F the right moving part of S_G is same as the left moving part for type IIB and the fields take opposite space-time chirality for type IIA. For definiteness we shall hereafter consider only type IIB string theory. The pure spinor constraints are given by,

$$\lambda^\alpha(z)\bar\gamma^\mu_{\alpha\beta}\lambda^\beta(z) = \tilde\lambda^\alpha(\bar z)\bar\gamma^\mu_{\alpha\beta}\tilde\lambda^\beta(\bar z) = 0\ . \tag{2.2}$$

Our notation for the gamma matrices can be found in appendix A. Because of these pure spinor constraints there are actually 11 independent fields instead of 16 and therefore S_G has central charges $c_G = \tilde c_G = 22$ (see [25] for a recent covariant computation of this central charge), instead of 32. This makes the total central charge of S zero. The local gauge transformations corresponding to the above constraints are given by,

$$\begin{aligned} \delta\lambda^\alpha(z) &= 0\ , &\quad \delta\tilde\lambda^\alpha(\bar z) &= 0\ ,\\ \delta w_\alpha(z) &= \Lambda_\mu(z)\bar\gamma^\mu_{\alpha\beta}\lambda^\beta(z)\ , &\quad \delta\tilde w_\alpha(\bar z) &= \tilde\Lambda_\mu(\bar z)\bar\gamma^\mu_{\alpha\beta}\tilde\lambda^\beta(\bar z)\ , \end{aligned} \tag{2.3}$$

which reduce the degrees of freedom of w_α and $\tilde w_\alpha$. As a result only the gauge invariant operators $\lambda^\alpha(z)w_\alpha(z)$ and $\lambda^\alpha(z)\bar\gamma^{\mu\nu}{}_\alpha{}^\beta w_\beta(z)$ (and similarly for the right moving sector) can appear in the construction of physical states. Space-time supersymmetry and BRST currents are given by $(\alpha' = 2)$,

$$\begin{aligned} q_\alpha &= p_\alpha + \frac{1}{2}(\bar\gamma^\mu\theta)_\alpha\partial X_\mu + \frac{1}{24}(\bar\gamma^\mu\theta)_\alpha(\theta\bar\gamma\partial\theta)\ ,\\ j_B &= \lambda^\alpha d_\alpha\ ,\quad d_\alpha = p_\alpha - \frac{1}{2}(\bar\gamma^\mu\theta)_\alpha\partial X_\mu - \frac{1}{8}(\bar\gamma^\mu\theta)_\alpha(\theta\bar\gamma_\mu\partial\theta)\ , \end{aligned} \tag{2.4}$$

and similarly for the right moving components.

Covariant quantization of the above CFT is not straightforward due to the pure spinor constraint. Nevertheless all the allowed states should form a subspace of the unconstrained Hilbert space which corresponds to the completely free theory. Performing the usual mode expansion with periodic boundary conditions for the world-sheet fields and quantizing the free theory one gets the following nontrivial commutation relations,

$$[\lambda^\alpha_m, w_{\beta,n}] = \delta^\alpha{}_\beta\delta_{m+n}\ ,\quad \{\theta^\alpha_m, p_{\beta,n}\} = \delta^\alpha{}_\beta\delta_{m+n}\ ,\quad m,n\in\mathbb{Z}\ , \tag{2.5}$$

and similarly for the right moving sector. The full Hilbert space is obtained by applying the negative modes freely on the ground states. Defining the states $|0\rangle$ and $|\hat 0\rangle$ in the following way,

$$\left.\begin{matrix}\lambda^\alpha_n\\ \\ \theta^\alpha_n\end{matrix}\right\}|0\rangle = 0\ ,\ \forall\alpha, n>0\ ,\qquad \left.\begin{matrix}w_{\alpha,n}\\ \\ p_{\alpha,n}\end{matrix}\right\}|0\rangle = 0\ ,\ \forall\alpha, n\ge 0\ ,$$

$$\left.\begin{array}{l}\lambda_n^\alpha \\ \\ \theta_n^\alpha\end{array}\right\}|\hat{0}\rangle = 0\ ,\ \forall\alpha, n\geq 0\ , \qquad \left.\begin{array}{l}w_{\alpha,n} \\ \\ p_{\alpha,n}\end{array}\right\}|\hat{0}\rangle = 0\ ,\ \forall\alpha, n>0\ , \tag{2.6}$$

all the ground states having the same L_0 eigenvalue can be obtained by either applying λ_0^α and θ_0^α repeatedly on $|0\rangle$ or applying $w_{\alpha,0}$ and $p_{\alpha,0}$ repeatedly on $|\hat{0}\rangle$. Since the ghost sector is bosonic the number of ground states corresponding to this sector is actually infinite. The ground states of the $(p_\alpha, \theta^\alpha)$ system, along with those of the bosonic matter, construct the $d = 9+1,\ \mathcal{N} = 1$ superspace.

Assigning the ghost numbers $(g, \tilde{g})$ to various fields in the following way: $\lambda^\alpha \to (1,0)$, $w_\alpha \to (-1,0)$, $\tilde{\lambda}^\alpha \to (0,1)$, $\tilde{w}_\alpha \to (0,-1)$ and others $\to (0,0)$, a physical on-shell vertex operator $V(z,\bar{z})$ is defined to be a $(1,1)$ operator such that,

$$Q_B|V\rangle = 0\ , \quad \tilde{Q}_B|V\rangle = 0\ , \tag{2.7}$$

where $Q_B = \oint \frac{dz}{2\pi i} j_B(z)$ (similarly for $\tilde{Q}_B$) and $|V\rangle = V(0,0)|0\rangle \otimes \widetilde{|0\rangle}$. Clearly the linearized gauge transformation is given by, $|\delta V\rangle = Q_B|\Phi\rangle + \tilde{Q}_B|\tilde{\Phi}\rangle$ for any ghost number $(0,1)$ and $(1,0)$ operators Φ and $\tilde{\Phi}$ respectively. In particular, the massless vertex operators are given by,

$$\begin{aligned}
\text{NS-NS}: &\ a^{(\mu}(z)\tilde{a}^{\nu)}(\bar{z})e^{ik.X}(z,\bar{z})\ , \\
\text{NS-R}: &\ a^{\mu}(z)\tilde{\chi}_\alpha(\bar{z})e^{ik.X}(z,\bar{z})\ , \\
\text{R-NS}: &\ \chi_\alpha(z)\tilde{a}^{\mu}(\bar{z})e^{ik.X}(z,\bar{z})\ , \\
\text{R-R (field strength)}: &\ \chi_\alpha(z)\tilde{\chi}_\beta(\bar{z})e^{ik.X}(z,\bar{z})\ ,
\end{aligned} \tag{2.8}$$

where, to the lowest order in the θ-expansion [5],[6]

$$a^\mu(z) = \lambda(z)\bar{\gamma}^\mu\theta(z)\ , \quad \chi_\alpha(z) = (\lambda(z)\bar{\gamma}^\mu\theta(z))(\bar{\gamma}_\mu\theta(z))_\alpha\ , \tag{2.9}$$

and similarly for the right moving operators. The zero modes saturation rule suggested by Berkovits [3] can be obtained by choosing a particular out-going ground state in the unconstrained theory [26],

$$\begin{aligned}
&\langle(\lambda_0\bar{\gamma}^\mu\theta_0)(\lambda_0\bar{\gamma}^\nu\theta_0)(\lambda_0\bar{\gamma}^\rho\theta_0)(\theta_0\bar{\gamma}_{\mu\nu\rho}\theta_0)\rangle_{Berkovits} \\
&= \langle\Omega|(\lambda_0\bar{\gamma}^\mu\theta_0)(\lambda_0\bar{\gamma}^\nu\theta_0)(\lambda_0\bar{\gamma}^\rho\theta_0)(\theta_0\bar{\gamma}_{\mu\nu\rho}\theta_0)|0\rangle = 1\ ,
\end{aligned} \tag{2.10}$$

where,

$$|\Omega\rangle = \frac{1}{c}(w_0\gamma^\mu p_0)(w_0\gamma^\nu p_0)(w_0\gamma^\rho p_0)(p_0\gamma_{\mu\nu\rho}p_0)|\hat{0}\rangle\ , \tag{2.11}$$

c being a numerical constant chosen properly to satisfy the second line of eq. (2.10). Here we adopt the following notation: $\langle\cdots\rangle_{Berkovits}$ refers to a correlation function computed in the actual constrained CFT. Any other inner product will be computed in the free CFT. Notice that according to the convention of Chesterman [26], the ghost numbers for the states $|0\rangle$ and $|\hat{0}\rangle$ are 8 and -8 respectively. Following the same convention we can find the ghost number of any given state in this CFT.

[6] Although the vertex operators are BRST invariant only when the full θ-expansions are considered, the leading order terms will suffice for our computations.

3. Boundary conditions and boundary states

Here we concentrate only on the combined fermionic matter and bosonic ghost part of the CFT as the bosonic matter part is well understood. As has been discussed before, we shall study the open string boundary conditions and the corresponding boundary states in the unconstrained CFT.[7] The traditional method of finding these boundary conditions is to take variation of the world-sheet action with respect to the basic fields and then set the boundary term to zero. In our case these conditions give the following equations on the upper half plane (UHP),

$$\left.\begin{aligned} w_\alpha(z)\delta\lambda^\alpha(z) &= \tilde{w}_\alpha(\bar{z})\delta\tilde{\lambda}^\alpha(\bar{z}) \ , \\ p_\alpha(z)\delta\theta^\alpha(z) &= \tilde{p}_\alpha(\bar{z})\delta\theta^\alpha(\bar{z}) \ , \end{aligned}\right\} \text{at } z = \bar{z} \ . \tag{3.1}$$

As was pointed out in [22], although the BPS boundary conditions can be easily obtained from the above conditions, the ones corresponding to non-BPS D-branes are not straightforward. We shall discuss both the cases below.

3.1 BPS D-branes

Let us consider a type IIB BPS Dp-brane ($p =$ odd) aligned along $x^0, x^1, \cdots, x^p$. Introducing the column vectors,

$$U^\alpha(z) = \begin{pmatrix} \lambda^\alpha(z) \\ \theta^\alpha(z) \end{pmatrix} \ , \quad V_\alpha(z) = \begin{pmatrix} w_\alpha(z) \\ p_\alpha(z) \end{pmatrix} \ , \tag{3.2}$$

and similarly for the right moving sector, the open string boundary conditions which satisfy eqs. (3.1) can be written as,

$$U^\alpha(z) = \eta(M^S)^\alpha{}_\beta \tilde{U}^\beta(\bar{z}) \ , \quad V_\alpha(z) = -\eta(\bar{M}^S)_\alpha{}^\beta \tilde{V}_\beta(\bar{z}) \ , \quad \text{at } z = \bar{z} \ , \tag{3.3}$$

where $\eta = \pm 1$ which correspond to brane and anti-brane respectively. The spinor matrices represent a set of reflections along the Neumann directions in the following way,

$$M^S = \gamma^{01\cdots p} \ , \quad \bar{M}^S = \bar{\gamma}^{01\cdots p} \ . \tag{3.4}$$

The multi-indexed gamma matrices are defined in appendix A. For the lorentzian D-branes that we are considering here,

$$\bar{M}^S(M^S)^T = (\bar{M}^S)^T M^S = -\mathbb{1}_{16} \ . \tag{3.5}$$

These matrices also satisfy the following relations,

$$\begin{aligned} M^S\gamma^\mu(M^S)^T &= -(M^V)^\mu{}_\nu\gamma^\nu \ , \quad (M^S)^T\bar{\gamma}^\mu M^S = -(M^V)^\mu{}_\nu\bar{\gamma}^\nu \ , \\ (\bar{M}^S)^T\gamma^\mu\bar{M}^S &= -(M^V)^\mu{}_\nu\gamma^\nu \ , \quad \bar{M}^S\bar{\gamma}^\mu(\bar{M}^S)^T = -(M^V)^\mu{}_\nu\bar{\gamma}^\nu \ , \end{aligned} \tag{3.6}$$

[7] Although the unconstrained CFT is not relevant to any string theory and therefore there does not exist any open string interpretation, we shall still continue to use this terminology by the abuse of language.

where M^V is the vector representation of the set of reflections along the Neumann directions,

$$(M^V)^\mu{}_\nu = \epsilon_{(\mu)}\delta^\mu{}_\nu \ , \quad \epsilon_{(\mu)} = \begin{cases} -1 \ \mu = 0, 1, \cdots, p, \\ +1 \ \mu = p+1, \cdots, 9 \ , \end{cases} \tag{3.7}$$

The boundary conditions for the supersymmetry and BRST currents turn out to be,

$$q_\alpha(z) = -\eta(\bar{M}^S)_\alpha{}^\beta \tilde{q}_\beta(\bar{z}) \ , \quad j_B(z) = \tilde{j}_B(\bar{z}) \ , \quad \text{at } z = \bar{z} \ . \tag{3.8}$$

The boundary state for such a D-brane situated at the origin of the transverse directions is given by,

$$|\mathrm{BPS}, p, \eta\rangle = \mathcal{N}_p \int d\vec{k}_\perp \exp\left(\sum_{n\geq 1} \frac{1}{n} \alpha_{\mu,-n} (M^V)^\mu{}_\nu \tilde{\alpha}^\nu_{-n}\right) |\vec{k}_\perp\rangle \otimes |F, p, \eta\rangle \ , \tag{3.9}$$

where $\mathcal{N}_p$ is a normalization constant proportional to the D-brane tension, $|\vec{k}_\perp\rangle$ is the bosonic Foch vacuum with momentum $\vec{k}_\perp$ along the transverse directions, the exponential factor is the usual bosonic oscillator part [21] and $|F, p, \eta\rangle$ is the combined fermionic matter and ghost part which satisfies the following closed string gluing conditions obtained from the boundary conditions (3.3),

$$\left.\begin{array}{l} U^\alpha_n - \eta(M^S)^\alpha{}_\beta \tilde{U}^\beta_{-n} \\ \\ V_{\alpha,n} - \eta(\bar{M}^S)_\alpha{}^\beta \tilde{V}_{\beta,-n} \end{array}\right\} |F, p, \eta\rangle = 0 \ , \quad \forall n \in \mathbb{Z} \ . \tag{3.10}$$

The solution is given by,

$$|F, p, \eta\rangle = \exp\left[-\eta \sum_{n\geq 1} \left(U^{\alpha T}_{-n} (\bar{M}^S)_\alpha{}^\beta \sigma_3 \tilde{V}_{\beta,-n} - \tilde{U}^{\alpha T}_{-n} (M^{ST})_\alpha{}^\beta \sigma_3 V_{\beta,-n}\right)\right] |F, p, \eta\rangle_0 \ , \tag{3.11}$$

where $\sigma_3 = \mathrm{diag}(1, -1)$ is the third Pauli matrix and the zero modes part $|F, p, \eta\rangle_0$ can be given the following two different forms:

$$\begin{aligned} |F, p, \eta\rangle_0 &= \exp\left(-\eta U^{\alpha T}_0 (\bar{M}^S)_\alpha{}^\beta \sigma_3 \tilde{V}_{\beta,0}\right) |0\rangle \otimes \widetilde{|\hat{0}\rangle} \ , \qquad \mathrm{I} \ , \\ &= \exp\left(\eta \tilde{U}^{\alpha T}_0 (M^{ST})_\alpha{}^\beta \sigma_3 V_{\beta,0}\right) |\hat{0}\rangle \otimes \widetilde{|0\rangle} \ , \qquad \mathrm{II} \ . \end{aligned} \tag{3.12}$$

We shall see in section 4.1 that both the above two forms can be used to compute one-point functions of the supergravity modes.

3.2 Non-BPS D-branes

Non-BPS D-branes in light-cone Green-Schwarz formalism have been studied in [27, 28, 22]. In particular, the covariant open string boundary conditions were found in [22]. Generalizing this work to any manifestly supersymmetric formalism we may write down the following general rules for finding the non-BPS boundary conditions on UHP,

1. Given a set of all the left moving world-sheet fields that are in space-time spinor representation, pair them up in all possible ways to form bi-local operators such as $A^\alpha(z)B^\beta(w)$, $A^\alpha(z)B_\beta(w)$, $A_\alpha(z)B^\beta(w)$ or $A_\alpha(z)B_\beta(w)$.

2. Expand them using Fiertz identities summarized in eqs. (A.8). These expansions will contain bi-local operators in tensor representations only.[8]

3. Equate the left and right moving bi-local operators in tensor representations on the real line by twisting them by the reflection matrix in vector representation.

In the present case we use the above rules to obtain the following open string boundary conditions in the unconstrained CFT:

$$\left.\begin{aligned} U^\alpha(z)U^{\beta T}(w) &= \mathcal{M}^{\alpha\beta}_{\gamma\delta}\tilde{U}^\gamma(\bar{z})\tilde{U}^{\delta T}(\bar{w}) \ , \\ U^\alpha(z)V^T_\beta(w) &= \mathcal{M}^{\alpha}{}_{\beta\gamma}{}^{\delta}\tilde{U}^\gamma(\bar{z})\tilde{V}^T_\delta(\bar{w}) \ , \\ V_\alpha(z)V^T_\beta(w) &= \mathcal{M}^{\gamma\delta}_{\alpha\beta}\tilde{V}_\gamma(\bar{z})\tilde{V}^T_\delta(\bar{w}) \ , \end{aligned}\right\} \text{at } z=\bar{z}, w=\bar{w} \ , \tag{3.13}$$

where the coupling matrices are given by (the relevant Fiertz identities are listed in appendix A),

$$\begin{aligned} \mathcal{M}^{\alpha\beta}_{\gamma\delta} &= -\Bigg[\frac{1}{16}\gamma^{\alpha\beta}_{\mu}(M^V)^{\mu}{}_{\nu}\bar{\gamma}^{\nu}_{\gamma\delta} + \frac{1}{16\times 3!}\gamma^{\alpha\beta}_{\mu_1\cdots\mu_3}(M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_3}{}_{\nu_3}\bar{\gamma}^{\nu_1\cdots\nu_3}_{\gamma\delta} \\ &\quad + \frac{1}{16\times 5!}\sum_{\mu_1,\cdots,\mu_5\in\mathcal{K}}\gamma^{\alpha\beta}_{\mu_1\cdots\mu_5}(M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_5}{}_{\nu_5}\bar{\gamma}^{\nu_1\cdots\nu_5}_{\gamma\delta}\Bigg] \ , \\ \mathcal{M}^{\alpha}{}_{\beta\gamma}{}^{\delta} &= \frac{1}{16}\delta^{\alpha}{}_{\beta}\delta_{\gamma}{}^{\delta} + \frac{1}{16\times 2!}\gamma_{\mu_1\mu_2}{}^{\alpha}{}_{\beta}(M^V)^{\mu_1}{}_{\nu_1}(M^V)^{\mu_2}{}_{\nu_2}\bar{\gamma}^{\nu_1\nu_2}{}_{\gamma}{}^{\delta} \\ &\quad + \frac{1}{16\times 4!}\gamma_{\mu_1\cdots\mu_4}{}^{\alpha}{}_{\beta}(M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_4}{}_{\nu_4}\bar{\gamma}^{\nu_1\cdots\nu_4}{}_{\gamma}{}^{\delta} \ . \end{aligned} \tag{3.14}$$

The summation convention for the repeated indices has been followed for all the terms in the above two equations except for the last term of the first equation. The sum over the five vector indices $\mu_1\cdots\mu_5$ has been explicitly restricted to a set $\mathcal{K}$ which is defined as follows. We divide the set of all possible sets of five indices $\{\{\mu_1,\cdots,\mu_5\}|\mu_i = 0,\cdots,9\}$ into two subsets of equal order, namely $\mathcal{K}$ and $\mathcal{K}_\mathcal{D}$ such that for every element $\{\mu_1,\cdots,\mu_5\}\in\mathcal{K}$ there exists a dual element $\{\mu_1,\cdots,\mu_5\}_D = \{\nu_1,\cdots,\nu_5\}\in\mathcal{K}_\mathcal{D}$ such that, $\epsilon^{\mu_1\cdots\mu_5\nu_1\cdots\nu_5}\neq 0$. A similar restriction is intimately related to the basis construction in light-cone Green-Schwarz formalism [22]. Using the properties (A.5) one can argue that replacing the restricted summation by a free summation would lead to zero for that particular term when M^V corresponds to a non-BPS D-brane. In that case eqs. (3.13) will not be invertible. As argued in [22], since the boundary conditions (3.13) are bi-local, one can take variation of fields independently at the two points. Using this one can easily show that eqs. (3.1) are satisfied. The supersymmetry current in eq. (2.4), being space-time fermionic, do not satisfy a linear

[8] As pointed out in [22], there is a subtlety involving the self-dual tensor operator in this expansion which needs to be taken care of. We shall do this explicitly below.

boundary condition. By using covariance one can argue that it satisfies the same boundary condition as p_α which can be read out from the last equation in (3.13). Absence of a linear boundary condition such as in eq. (3.8) implies that all the supersymmetries are broken. But the BRST current does satisfy the same condition as in eq. (3.8).

It may seem difficult to obtain the boundary states corresponding to the above boundary conditions. But we follow a trick discussed in [22] to achieve this. By a "bosonization and refermionization" method we first define a new set of right moving fields $\bar{U}^\alpha(\bar{z})$ and $\bar{V}_\alpha(\bar{z})$ in the following way,

$$\begin{aligned}
\bar{U}^\alpha(\bar{z})\bar{\gamma}^{\mu}_{\alpha\beta}\bar{U}^{\beta T}(\bar{w}) &= -(M^V)^{\mu}{}_{\nu}\tilde{U}^\alpha(\bar{z})\bar{\gamma}^{\nu}_{\alpha\beta}\tilde{U}^{\beta T}(\bar{w}) \ , \\
\bar{U}^\alpha(\bar{z})\bar{\gamma}^{\mu_1\cdots\mu_3}_{\alpha\beta}\bar{U}^{\beta T}(\bar{w}) &= -(M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_3}{}_{\nu_3}\tilde{U}^\alpha(\bar{z})\bar{\gamma}^{\nu_1\cdots\nu_3}_{\alpha\beta}\tilde{U}^{\beta T}(\bar{w}) \ , \\
\bar{U}^\alpha(\bar{z})\bar{\gamma}^{\mu_1\cdots\mu_5}_{\alpha\beta}\bar{U}^{\beta T}(\bar{w}) &= -\begin{cases} (M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_5}{}_{\nu_5}\tilde{U}^\alpha(\bar{z})\bar{\gamma}^{\nu_1\cdots\nu_5}_{\alpha\beta}\tilde{U}^{\beta T}(\bar{w}) \ , \ \mu_1,\cdots,\mu_5 \in \mathcal{K} \ , \\ -(M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_5}{}_{\nu_5}\tilde{U}^\alpha(\bar{z})\bar{\gamma}^{\nu_1\cdots\nu_5}_{\alpha\beta}\tilde{U}^{\beta T}(\bar{w}) \ , \ \mu_1,\cdots,\mu_5 \in \mathcal{K}_{\mathcal{D}} \ , \end{cases}
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
\bar{V}_\alpha(\bar{z})\gamma^{\mu\alpha\beta}\bar{V}^T_\beta(\bar{w}) &= -(M^V)^{\mu}{}_{\nu}\tilde{V}_\alpha(\bar{z})\gamma^{\nu\alpha\beta}\tilde{V}^T_\beta(\bar{w}) \ , \\
\bar{V}_\alpha(\bar{z})\gamma^{\mu_1\cdots\mu_3\alpha\beta}\bar{V}^T_\beta(\bar{w}) &= -(M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_3}{}_{\nu_3}\tilde{V}_\alpha(\bar{z})\bar{\gamma}^{\nu_1\cdots\nu_3\alpha\beta}\tilde{V}^T_\beta(\bar{w}) \ , \\
\bar{V}_\alpha(\bar{z})\gamma^{\mu_1\cdots\mu_5\alpha\beta}\bar{V}^T_\beta(\bar{w}) &= -\begin{cases} (M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_5}{}_{\nu_5}\tilde{V}_\alpha(\bar{z})\gamma^{\nu_1\cdots\nu_5\alpha\beta}\tilde{V}^T_\beta(\bar{w}) \ , \ \mu_1,\cdots,\mu_5 \in \mathcal{K} \ , \\ -(M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_5}{}_{\nu_5}\tilde{V}_\alpha(\bar{z})\gamma^{\nu_1\cdots\nu_5\alpha\beta}\tilde{V}^T_\beta(\bar{w}) \ , \ \mu_1,\cdots,\mu_5 \in \mathcal{K}_{\mathcal{D}} \ , \end{cases}
\end{aligned} \tag{3.16}$$

$$\begin{aligned}
\bar{U}^\alpha(\bar{z})\bar{V}^T_\alpha(\bar{w}) &= \tilde{U}^\alpha(\bar{z})\tilde{V}^T_\alpha(\bar{w}) \ , \\
\bar{U}^\alpha(\bar{z})\bar{\gamma}^{\mu_1\mu_2}{}_{\alpha}{}^{\beta}\bar{V}^T_\beta(\bar{w}) &= (M^V)^{\mu_1}{}_{\nu_1}(M^V)^{\mu_2}{}_{\nu_2}\tilde{U}^\alpha(\bar{z})\bar{\gamma}^{\nu_1\nu_2}{}_{\alpha}{}^{\beta}\tilde{V}^T_\beta(\bar{w}) \ , \\
\bar{U}^\alpha(\bar{z})\bar{\gamma}^{\mu_1\cdots\mu_4}{}_{\alpha}{}^{\beta}\bar{V}^T_\beta(\bar{w}) &= (M^V)^{\mu_1}{}_{\nu_1}\cdots(M^V)^{\mu_4}{}_{\nu_4}\tilde{U}^\alpha(\bar{z})\bar{\gamma}^{\nu_1\cdots\nu_4}{}_{\alpha}{}^{\beta}\tilde{V}^T_\beta(\bar{w}) \ .
\end{aligned} \tag{3.17}$$

Notice the definitions of the anti-self-dual and self-dual operators $\bar{U}^\alpha(\bar{z})\bar{\gamma}^{\mu_1\cdots\mu_5}_{\alpha\beta}\bar{U}^{\beta T}(\bar{w})$ and $\bar{V}_\alpha(\bar{z})\gamma^{\mu_1\cdots\mu_5\alpha\beta}\bar{V}^T_\beta(\bar{w})$ in eqs. (3.15) and (3.16) respectively. There is a sign difference between the cases when the set of indices belong to $\mathcal{K}$ and $\mathcal{K}_{\mathcal{D}}$. This is because for a non-BPS D-brane M^V represents a set of odd number of reflections under which a self-dual tensor transforms to an anti-self-dual tensor and vice-versa. In terms of these new fields the boundary conditions in (3.13) take the following simpler form,

$$\left.\begin{aligned}
U^\alpha(z)U^{\beta T}(w) &= \bar{U}^\alpha(\bar{z})\bar{U}^{\beta T}(\bar{w}) \ , \\
U^\alpha(z)V^T_\beta(w) &= \bar{U}^\alpha(\bar{z})\bar{V}^T_\beta(\bar{w}) \ , \\
V_\alpha(z)V^T_\beta(w) &= \bar{V}_\alpha(\bar{z})\bar{V}^T_\beta(\bar{w}) \ ,
\end{aligned}\right\} \text{at } z=\bar{z}, w=\bar{w} \ . \tag{3.18}$$

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The closed string gluing conditions obtained from these boundary conditions are given by,

$$\left.\begin{aligned} (U^\alpha_m U^{\beta T}_n - \bar{U}^\alpha_{-m}\bar{U}^{\beta T}_{-n}) \\ (U^\alpha_m V^T_{\beta,n} + \bar{U}^\alpha_{-m}\bar{V}^T_{\beta,-n}) \\ (V_{\alpha,m}V^T_{\beta,n} - \bar{V}_{\alpha,-m}\bar{V}^T_{\beta,-n}) \end{aligned}\right\} |F,p\rangle = 0 \ , \quad \forall m,n \in \mathbb{Z} \ . \tag{3.19}$$

One can show that this is precisely the gluing conditions satisfied by the NS-NS part of the D9 boundary state, which is given by eqs. (3.11), (3.12) for $M^S = \mathbb{1}_{16}$ and $\bar{M}^S = -\mathbb{1}_{16}$, with the right moving oscillators replaced by the corresponding barred oscillators. Notice that there is no replacement for the right moving ground state in either $|0\rangle \otimes \widetilde{|\hat{0}\rangle}$ or $|\hat{0}\rangle \otimes \widetilde{|0\rangle}$. This is because $\widetilde{|0\rangle}$ and $\widetilde{|\hat{0}\rangle}$ are the two ground states that satisfy the same equations as (2.6) with all the oscillators replaced by the corresponding barred oscillators. Since it is the NS-NS part of the state in (3.11, 3.12) that is relevant, any term in the expansion of the non-BPS boundary state (written in terms of the barred variables) can easily be translated back in terms of the original right moving variables using the relations (3.15), (3.16) and (3.17). Changing the definition of the barred variables suitably the non-BPS boundary state can be given the form of the NS-NS part of any BPS boundary state discussed in the previous subsection [22].

4. Computing physical quantities

Given the boundary states in the previous section one would obviously wonder how to use them to compute physical quantities. It is not a priori clear how to do such computations. Hoping that the present method of dealing with boundary states really works, our approach will be to find prescriptions for such computations. Below we shall discuss two such issues namely, the closed string sources and D-brane interaction.

4.1 Sources for massless closed string modes

A D-brane acts like a source for various closed string modes. The strength of these sources can be computed either by using the boundary state or the boundary conformal field theory. Without going into much technical details the NSR computation can be described as follows: Given a closed string state $|\psi\rangle$ there exists a corresponding conjugate state $\langle\psi^{(C)}|$ such that the strength is given by saturating the conjugate state with the boundary state. The same result is obtained in the BCFT by computing the disk one-point function of $\psi^{(C)}$ where the vertex operator is inserted at the origin of the unit disk. Given a closed string state the correct conjugate operator has to be found by properly satisfying the ghost and superghost zero mode saturation rules. Although the details of the computation should look different in pure spinor formalism, the same general features are expected to be realized. Here we suggest the conjugate vertex operators for the massless closed string states and show how the computation goes through in the present situation. We begin with the boundary state computation and at the end indicate how to compute the disk one-point functions. We focus only on the combined fermionic matter and bosonic ghost part as it is known how to deal with the rest of the CFT. Let us first define the following operators,

$$a^\mu = \lambda_0 \bar\gamma^\mu \theta_0 \ , \ \ \chi_\alpha = (\lambda_0 \bar\gamma^\mu \theta_0)(\bar\gamma_\mu \theta_0)_\alpha \ , \tag{4.1}$$

and similarly for the right moving sector, so that the relevant parts of the supergravity states are given by (to the lowest order in θ-expansion),

$$\begin{aligned} \text{NS-NS:} \quad & |V^{\mu\nu}\rangle = a^\mu \tilde a^\nu |0\rangle \otimes \widetilde{|0\rangle} \ , \\ \text{NS-R:} \quad & |\Psi^\mu_\alpha\rangle = a^\mu \tilde\chi_\alpha |0\rangle \otimes \widetilde{|0\rangle} \ , \\ \text{R-NS:} \quad & |\bar\Psi^\mu_\alpha\rangle = \chi_\alpha \tilde a^\mu |0\rangle \otimes \widetilde{|0\rangle} \ , \\ \text{R-R (field strength):} \quad & |F_{\alpha\beta}\rangle = \chi_\alpha \tilde\chi_\beta |0\rangle \otimes \widetilde{|0\rangle} \ . \end{aligned} \tag{4.2}$$

Then we define,

$$a^{(C)}_\mu \propto (\lambda_0 \bar\gamma^\nu \theta_0)(\lambda_0 \bar\gamma^\rho \theta_0)(\theta_0 \bar\gamma_{\mu\nu\rho} \theta_0) \ , \ \ \chi^{(C)\alpha} \propto (\lambda_0 \bar\gamma^\mu \theta_0)(\lambda_0 \bar\gamma^\nu \theta_0)(\theta_0 \bar\gamma_{\mu\nu})^\alpha \ . \tag{4.3}$$

The proportionality constants are determined by demanding that the above operators are conjugate to the operators in eqs. (4.1) in the following sense,

$$\langle a^{(C)}_\mu a^\nu \rangle_{Berkovits} = \langle \Omega | a^{(C)}_\mu a^\nu |0\rangle = \delta_\mu^{\ \nu} \ , \quad \langle \chi^{(C)\alpha} \chi_\beta \rangle_{Berkovits} = \langle \Omega | \chi^{(C)\alpha} \chi_\beta |0\rangle = \delta^\alpha_{\ \beta} \tag{4.4}$$

The operators defined in eqs. (4.3) appear at the lowest orders in θ-expansions of the two elements of BRST cohomology at ghost number two. The higher order terms are irrelevant for the present purpose as they do not contribute to the above inner products. These BRST elements correspond to what are called antifields associated to gluon (a^μ) and gluino (χ_α) respectively[29]. Defining the conjugate states corresponding to the NS-NS sector in the following way,

$$_I\langle V^{(C)}_{\mu\nu}| = \widetilde{\langle 0|} \tilde a_\mu \otimes \langle \Omega | a^{(C)}_\nu \ , \tag{4.5}$$

we get using form I in eq. (3.12),[9]

$$_I\langle V^{(C)}_{\mu\nu}|F, p, \eta\rangle_0 = -\eta_{\mu\rho}(M^V)^\rho_{\ \nu} \ , \tag{4.6}$$

which is a desired result. Notice that the right-moving part of the dual state, namely $\widetilde{\langle 0|}\tilde a_\mu$ couples with the term in the expansion of the boundary state that is linear in both $\tilde w$ and $\tilde p$ whose left moving part is also linear in both λ and θ. This particular combination is properly saturated by the left moving part in the definition (4.5). The overall constant in the result (4.6) is not important as it can always be absorbed in the definition (4.5). Notice that we could also define the dual state by interchanging left and right moving objects in eq. (4.5), namely,

$$_{II}\langle V^{(C)}_{\mu\nu}| = -\widetilde{\langle \Omega|} \tilde a^{(C)}_\mu \otimes \langle 0 | a_\nu . \tag{4.7}$$

[9] We follow a convention where $\tilde\theta^\alpha$ and $\tilde p_\alpha$ pick up a minus sign while passing through the spin field corresponding to the state $|0\rangle$.

In that case we need to use form II in eq. (3.12) in order to get the correct result as in eq. (4.6).

Let us now turn to the R-R states $|F_{\alpha\beta}\rangle$. One would expect for the corresponding conjugate states to have an index structure: $\langle F^{(C)\alpha\beta}|$. But since each of the left and right moving parts of $|F_{\alpha\beta}\rangle$ contributes a spinor index, the conjugate states, obtained by following the above procedure, have spinor indices of opposite chirality.

$$ {}_I\langle C_1^{(C)\alpha}{}_\beta| = \widetilde{\langle 0|}\tilde{\chi}_\beta \otimes \langle\Omega|\chi^{(C)\alpha} \ . \tag{4.8} $$

We would like to interpret these conjugate states to correspond to R-R potential rather than R-R field strength. Using form I of the boundary state one can show that the above state produces the following result which is consistent with this interpretation,

$$ {}_I\langle C_1^{(C)\alpha}{}_\beta|F,p,\eta\rangle_0 = \eta(M^S)^\alpha{}_\beta \ . \tag{4.9} $$

The subscript 1 in the above notation will now be explained. The R-R potential has been suggested in the literature to be given by the following state [30, 19],

$$ |C_\alpha{}^\beta\rangle = \left(\chi_\alpha\tilde{\lambda}_0^\beta + \lambda_0^\beta\tilde{\chi}_\alpha\right)|0\rangle \otimes \widetilde{|0\rangle} \ . \tag{4.10} $$

Given this one may wonder why the conjugate state in eq. (4.8) is entirely constructed out of χ and its conjugate. To answer this question one may consider the following state,

$$ {}_I\langle C_2^{(C)\alpha}{}_\beta| = \widetilde{\langle 0|}\tilde{\lambda}_0^\alpha \otimes \langle\Omega|\lambda_\beta^{(C)} \ , \tag{4.11} $$

where

$$ \lambda_\alpha^{(C)} \propto (\lambda_0\bar{\gamma}^\mu\theta_0)(\lambda_0\bar{\gamma}^\nu\theta_0)(\bar{\gamma}^\rho\theta_0)_\alpha(\theta_0\bar{\gamma}_{\mu\nu\rho}\theta_0) \ \text{,such that } \langle\lambda_\alpha^{(C)}\lambda^\beta\rangle_{Berkovits} = \delta_\alpha{}^\beta \ . \tag{4.12} $$

This gives the similar result,

$$ {}_I\langle C_2^{(C)\alpha}{}_\beta|F,p,\eta\rangle_0 = -\eta(\bar{M}^{S^T})^\alpha{}_\beta = \eta(M^{S^{-1}})^\alpha{}_\beta \ . \tag{4.13} $$

We may take the conjugate of the R-R potential in form I to be a linear combination of ${}_I\langle C_1^{(C)\alpha}{}_\beta|$ and ${}_I\langle C_2^{(C)\alpha}{}_\beta|$. There are alternative expressions for the dual states as well which are suitable for computing inner products with form II of the boundary state. These are given by,

$$ \begin{aligned} {}_{II}\langle C_1^{(C)\alpha}{}_\beta| &= \widetilde{\langle\Omega|}\tilde{\chi}^{(C)\alpha} \otimes \langle 0|\chi_\beta \ , \qquad {}_{II}\langle C_1^{(C)\alpha}{}_\beta|F,p,\eta\rangle_0 = \eta(M^{S^{-1}})^\alpha{}_\beta \ , \\ {}_{II}\langle C_2^{(C)\alpha}{}_\beta| &= \widetilde{\langle\Omega|}\tilde{\lambda}_\beta^{(C)} \otimes \langle 0|\lambda_0^\alpha \ , \qquad {}_{II}\langle C_2^{(C)\alpha}{}_\beta|F,p,\eta\rangle_0 = \eta(M^S)^\alpha{}_\beta \ . \end{aligned} \tag{4.14} $$

Because of the particular index structure of the R-NS and NS-R states $|\bar{\Psi}_\alpha^\mu\rangle$ and $|\Psi_\alpha^\mu\rangle$ respectively, it turns out that for each one of them a conjugate state corresponding to only one form can be constructed, namely,

$$ {}_I\langle\bar{\Psi}^{(C)\alpha}_\mu| = \widetilde{\langle 0|}\tilde{a}_\mu \otimes \langle\Omega|\chi^{(C)\alpha} \ , \qquad {}_{II}\langle\Psi^{(C)\alpha}_\mu| = \widetilde{\langle\Omega|}\tilde{\chi}^{(C)\alpha} \otimes \langle 0|a_\mu \ . \tag{4.15} $$

Vertex operator	Form I	Form II
$V^{(C)}_{\mu\nu}(\zeta,\bar{\zeta})$	$a^{(C)}_{\nu}(\zeta)\tilde{a}_{\mu}(\bar{\zeta})$	$a_{\nu}(\zeta)\tilde{a}^{(C)}_{\mu}(\bar{\zeta})$
$\Psi^{(C)\alpha}{}_{\mu}(\zeta,\bar{\zeta})$	-	$-a_{\mu}(\zeta)\tilde{\chi}^{(C)\alpha}(\bar{\zeta})$
$\bar{\Psi}^{(C)\alpha}{}_{\mu}(\zeta,\bar{\zeta})$	$\chi^{(C)\alpha}(\zeta)\tilde{a}_{\mu}(\bar{\zeta})$	-
$C_1^{(C)\alpha}{}_{\beta}(\zeta,\bar{\zeta})$	$\chi^{(C)\alpha}(\zeta)\tilde{\chi}_{\beta}(\bar{\zeta})$	$\chi_{\beta}(\zeta)\tilde{\chi}^{(C)\alpha}(\bar{\zeta})$
$C_2^{(C)\alpha}{}_{\beta}(\zeta,\bar{\zeta})$	$\lambda^{(C)}_{\beta}(\zeta)\tilde{\lambda}^{\alpha}(\bar{\zeta})$	$\lambda^{\alpha}(\zeta)\tilde{\lambda}^{(C)}_{\beta}(\bar{\zeta})$

Table 1: Massless closed string vertex operators to be used in the computation of disk one-point functions. $(\zeta,\bar{\zeta})$ denote the complex coordinates on a unit disk.

They can be shown to give zero inner product with the correct form of the boundary state,

$$
{}_{I}\langle\bar{\Psi}^{(C)\alpha}{}_{\mu}|F,p,\eta\rangle_0 = 0 \ , \qquad {}_{II}\langle\Psi^{(C)\alpha}{}_{\mu}|F,p,\eta\rangle_0 = 0 \ , \tag{4.16}
$$

which are also expected results.

The dual states used in the boundary state computation should directly give the vertex operators that need to be used in the disk one point functions. A list of those operators for the massless modes is given in table 1. To compute the disk one-point functions one may first use the conformal transformation: $z = i(1+\zeta)/(1-\zeta)$ to go from the unit disk to UHP.

$$
\langle\psi^{(C)}(0,0)\rangle^{Disk}_{Berkovits} = \langle\psi^{(C)}(i,-i)\rangle^{UHP}_{Berkovits} \ , \tag{4.17}
$$

where $\psi^{(C)}$ is a vertex operator in table 1. Notice that all these operators have conformal dimension zero. Given a form I vertex operator one can compute the right hand side of eq. (4.17) in the following way: First define the following operators on the *doubled* surface,

$$
\mathcal{U}^{\alpha}(u) = \begin{cases} U^{\alpha}(z)|_{z=u} \ , \\ \\ \eta(M^S)^{\alpha}{}_{\beta}\tilde{U}^{\beta}(\bar{z})|_{\bar{z}=u} \ , \end{cases} \qquad \mathcal{V}_{\alpha}(u) = \begin{cases} V_{\alpha}(z)|_{z=u} \ , & \Im u \geq 0 \ , \\ \\ -\eta(\bar{M}^S)_{\alpha}{}^{\beta}\tilde{V}_{\beta}(\bar{z})|_{\bar{z}=u} \ , & \Im u \leq 0 \ , \end{cases} \tag{4.18}
$$

then use them to convert the right hand side of eq. (4.17) to a holomorphic correlation function on the full plane.[10] For dealing with the form II vertex operators one follows the same procedure except that now instead of using the operators in eqs. (4.18) one uses the

[10]Notice that although the doubling trick (4.18) corresponds to boundary conditions (3.3) in the free CFT, it is being used to compute the correlator (4.17) in the constrained theory. This can be justified by the fact that due to covariance (4.18) gives the correct doubling for any physical operator in the constrained CFT that appears in (4.17).

following operators,

$$\tilde{\mathcal{U}}^\alpha(\bar{u}) = \begin{cases} \tilde{U}^\alpha(\bar{z})|_{\bar{z}=\bar{u}} \ , \\ \\ -\eta(\bar{M}^{S^T})^\alpha{}_\beta U^\beta(z)|_{z=\bar{u}} \ , \end{cases} \qquad \tilde{\mathcal{V}}_\alpha(\bar{u}) = \begin{cases} \tilde{V}_\alpha(\bar{z})|_{\bar{z}=\bar{u}} \ , & \Im u \geq 0 \ , \\ \\ \eta(M^{s^T})_\alpha{}^\beta V_\beta(z)|_{z=\bar{u}} \ , & \Im u \leq 0 \ . \end{cases} \tag{4.19}$$

to convert the right hand side of (4.17) to an anti-holomorphic correlation function. Following this procedure one establishes the following relation in any relevant form,

$$\langle\psi^{(C)}|F,p,\eta\rangle_0 = \langle\psi^{(C)}(0,0)\rangle^{Disk}_{Berkovits} \ . \tag{4.20}$$

4.2 Comments on force computation

Interaction between D-branes using boundary states is an important computation to be understood in pure spinor formalism. This is also related to the understanding of open-closed duality. In NRS formalism we are aware of a simple gauge choice (Siegel gauge) in which the closed string propagator can be written in terms of the world-sheet hamiltonian so that the boundary state when evolved by this propagator forms the *cylinder diagram*. In the present case the similar gauge choice is not known and therefore it is not *a priory* clear how to go through this computation. But here we shall try to emphasize yet another feature of the pure spinor computation. In NSR formalism the associated ghost degrees of freedom give a certain background independent contribution which cancels two coordinates (light-cone) worth of contribution from the matter part to give the correct physical result. From the boundary states discussed in the previous section it seems difficult to obtain such a background independent contribution. Below we shall demonstrate this feature by focusing on the simplest computation of this type namely, the long range NS-NS force between two parallel BPS D-branes.

Let us first review how the computation goes through in NSR formalism. The relevant part of the boundary state is the NS-NS part of a BPS Dp-brane situated at $\vec{y}_\perp$ along the transverse directions [21],

$$\begin{aligned} |NSNS,p,\vec{y}_\perp\rangle^{massless}_{NSR} \propto \int d\vec{k}_\perp e^{-i\vec{k}_\perp\cdot\vec{y}_\perp} \Big[&\eta_{\mu\rho}(M^V)^\rho{}_\nu \psi^\mu_{-1/2}\tilde{\psi}^\nu_{-1/2} \\ &-(\gamma_{-1/2}\tilde{\beta}_{-1/2} - \beta_{-1/2}\tilde{\gamma}_{-1/2})\Big]|\vec{k}_\perp, k_{||}=0\rangle_{NSR} \ , \end{aligned} \tag{4.21}$$

where the proportionality constant is linear in the tension of the D-brane, $\psi^\mu_{-1/2}$, $\tilde{\psi}^\mu_{-1/2}$, $\beta_{-1/2}$, $\tilde{\beta}_{-1/2}$, $\gamma_{-1/2}$, $\tilde{\gamma}_{-1/2}$ are the usual left and right moving fermionic matter and bosonic superghosts in NS sector and $|\vec{k}_\perp, k_{||}=0\rangle_{NSR}$ is the ghost number 3, picture number $(-1,-1)$ Foch vacuum with the indicated momenta,

$$|\vec{k}_\perp, k_{||}=0\rangle_{NSR} = (c_0+\tilde{c}_0)c_1\tilde{c}_1 e^{-(\phi+\tilde{\phi})}(0,0)e^{i\vec{k}_\perp\cdot\vec{X}}(0,0)|0\rangle \ . \tag{4.22}$$

Notice that the superghost oscillator part in eq. (4.21) is the ghost-dilaton which, combined with the trace of $\psi^\mu_{-1/2}\tilde{\psi}^\nu_{-1/2}$, constructs the dilaton state. We shall see how ghost-dilaton plays its role in producing the correct physical result for the long distance force. The long distance NS-NS force between two parallel D-branes Dp' and Dp situated at $\vec{y}'_\perp$ and $\vec{y}_\perp$ is obtained from the following amplitude,

$$\mathcal{A}_{NSNS} = \int_0^\infty dt \ {}^{massless}_{NSR}\langle NSNS, p', \vec{y}'_\perp| e^{-\pi t(L_0+\tilde{L}_0)} |NSNS, p, \vec{y}_\perp\rangle^{massless}_{NSR} \ , \quad (4.23)$$

where L_0 and $\tilde{L}_0$ are, as usual, the Virasoro zero modes. The result turns out to be,

$$\mathcal{A}_{NS-NS} = f(\vec{y}'_\perp, \vec{y}_\perp)\left[(10-2\nu)-2\right] \ , \quad (4.24)$$

where $f(\vec{y}'_\perp, \vec{y}_\perp)$ is a function whose details are not needed for our purpose. We are interested only in the numerical factor kept in the square bracket. ν is the number of Neumann-Dirichlet directions involved in this configuration. The background dependent part $(10-2\nu)$ comes from $\mathrm{Tr}(M'^V M^{V^T})$ contributed by the fermionic matter part of the state in eq. (4.21) whereas the background independent contribution -2 is provided by the ghost-dilaton state. Because of this contribution the force is zero for a supersymmetric configuration in which $\nu = 4$.

Let us now try to see how the same result might be reproduced in pure spinor formalism using the boundary state (3.9). The relevant part of the boundary state is,

$$|NSNS, p, \vec{y}_\perp\rangle^{massless}_{PS} \propto \frac{1}{2}\int d\vec{k}_\perp e^{-i\vec{k}_\perp\cdot\vec{y}_\perp}\Big[\exp\left(-U_0^{\alpha T}(\bar{M}^S)_\alpha{}^\beta\sigma_3\tilde{V}_{\beta,0}\right) + \exp\left(U_0^{\alpha T}(\bar{M}^S)_\alpha{}^\beta\sigma_3\tilde{V}_{\beta,0}\right)\Big]\, |\vec{k}_\perp, k_{||}=0\rangle\otimes|0\rangle\otimes\widetilde{|\hat{0}\rangle} \ , \quad (4.25)$$

Recall that only a finite number of states (which are in one-to-one correspondence with the physical states) appeared in eq. (4.21) for each momentum. But eq. (4.25) contains an infinite number of states. There are two basic sources for having lots of extra states in pure spinor formalism. The first one is due to the fact that the boundary state has been constructed in the gauge unfixed theory where the $d = 9+1$, $\mathcal{N} = 2$ superspace structure for the target space is manifest. This results in a lot of auxiliary fields, which in addition to the physical ones, appear in the boundary state in (4.25). Secondly, this boundary state has been constructed in the bigger Hilbert space of the unconstrained CFT. For the relevant computation one could suggest a prescription of first projecting the boundary state onto the ones corresponding to the physical states,

$$|NSNS, p, \vec{y}_\perp\rangle^{massless}_{PS} \rightarrow \int d\vec{k}_\perp e^{-i\vec{k}_\perp\cdot\vec{y}_\perp}\eta_{\mu\rho}(M^V)^\rho{}_\nu\,(\lambda_0\bar{\gamma}^\mu\theta_0)\,(\tilde{w}_0\gamma^\nu\tilde{p}_0)\, |\vec{k}_\perp, k_{||}=0\rangle\otimes|0\rangle\otimes\widetilde{|\hat{0}\rangle} \ . \quad (4.26)$$

and then evolving that by world-sheet time evolution. But because of the absence of an analog of ghost-dilaton in this formalism one does not get the background independent contribution of -2 in the square bracket of eq. (4.24). Absence of the ghost-dilaton in pure spinor formalism does not necessarily imply any inconsistency as far as the present computation is concerned. This is because we may think of this force computation simply as a problem in the space-time theory whose spectrum is correctly reproduced in pure spinor formalism. In this computation one is simply required to evaluate the tree-level Feynman diagram where all the NS-NS massless modes propagate between two sources representing the D-branes. The role of the boundary states is only to provide the strength of these sources. Extending this observation to higher levels one may suggest that the projected boundary state should actually be thought of as the NSR boundary state in old covariant quantization which gives correct sources for the space-time fields but is insufficient to organize the force computation keeping the open-closed duality manifest.

5. Discussion

We end by mentioning several interesting questions worth exploring in future work.

1. In this paper we constructed D-brane boundary states in the unconstrained CFT by relaxing the pure spinor constraint. This simply enables us to work in a larger Hilbert space so that all the inner product computations relevant to the pure spinor CFT can also be done here. Therefore these boundary states produce the correct results for all the closed string sources once the correct prescription is followed. Despite this advantage these boundary states are not suitable for computing the cylinder diagram with manifest open-closed duality. The problem, as explained in the previous section, can not be solved only by imposing the pure spinor constraint, but the theory has also to be gauge fixed at all mass levels. In order to compute the cylinder diagram with manifest open-closed duality one needs to project the boundary states constructed here onto a smaller Hilbert space such that both the pure spinor constraint and the removal of the gauge degrees of freedom is implemented. This has been explicitly done in [31] by constructing the physical Hilbert space through the DDF construction. Although this enables us to compute the cylinder diagram in pure spinor formalism, the projected boundary states are covariant only under the transverse $SO(8)$ part. Construction of the boundary states suitable for computing the cylinder diagram with full $SO(9,1)$ covariance is still an open question. Computation of the R-R force between two D-branes is not straightforward even in NSR formalism. Certain regularization method is involved in this computation to control the zero modes contribution coming from the bosonic superghosts [24]. A complete understanding of the covariant boundary states in pure spinor formalism has to solve all these subtle issues.

2. We have also discussed open string boundary conditions in the unconstrained CFT. Because of the manifest covariance these boundary conditions produce correct reflection property between the left and right moving parts of any closed string vertex operator that is allowed in the pure spinor superstrings. Therefore it should be possible to use these boundary conditions to compute disk scattering amplitudes. Here we have prescribed rules for computing disk one point functions for massless closed strings only. This needs to be extended to all possible disk amplitudes with arbitrary number of bulk and boundary insertions.

3. The open string boundary conditions for non-BPS D-branes have been suggested here by generalizing the previous work on light-cone GS formalism in [22]. It should be further investigated if this generalization gives sensible results. In particular, it would be interesting to see if the rules for computing disk amplitudes with arbitrary bulk insertions, as suggested in [22], also have sensible generalization to the present case.

4. In [22] it was shown that the bi-local boundary conditions on space-time fermions for a non-BPS D-brane give rise to two sectors in the open string spectrum. One is given by periodic boundary condition resulting in a Bose-Fermi degenerate spectrum same as that for a BPS D-brane. The other is given by anti-periodic boundary condition. This sector includes the tachyon and is responsible for the fermion doubling at the massless level. Following the same analysis in the pure spinor case also one gets the similar two sectors. Analysis for the periodic sector goes just like that for a BPS D-brane. The anti-periodic sector has recently been analyzed in [31] and all the physical open string states have been explicitly constructed through DDF construction. Despite this progress it is still unknown how to construct the tachyon vertex operator. This is because the spin fields of the pure spinor ghosts with anti-periodic boundary condition are not understood. The same problem also exists for multiple D-brane configurations at various angles. In this case the open strings going from one brane to another will have more general boundary conditions.

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A. Gamma matrices and Fiertz identities

We denote a 32-component SO(9,1)-spinor in the Weyl basis as $(\chi^\alpha, \xi_\alpha)$, where $\alpha = 1, \cdots, 16$. The 32-dimensional gamma matrices are given by,

$$\Gamma^\mu = \begin{pmatrix} 0 & \gamma^{\mu\alpha\beta} \\ \bar{\gamma}^\mu_{\alpha\beta} & 0 \end{pmatrix} , \quad \mu = 0, 1, \cdots, 9 , \tag{A.1}$$

such that,

$$(\gamma^\mu \bar{\gamma}^\nu + \gamma^\nu \bar{\gamma}^\mu)^\alpha{}_\beta = 2\eta^{\mu\nu}\delta^\alpha{}_\beta , \quad (\bar{\gamma}^\mu \gamma^\nu + \bar{\gamma}^\nu \gamma^\mu)_\alpha{}^\beta = 2\eta^{\mu\nu}\delta_\alpha{}^\beta . \tag{A.2}$$

The 16-dimensional gamma matrices are symmetric and give the following chirality matrix:

$$\Gamma = \Gamma^0 \Gamma^1 \cdots \Gamma^9 = \begin{pmatrix} \gamma^{01\cdots 9} & 0 \\ 0 & \bar{\gamma}^{01\cdots 9} \end{pmatrix} = \begin{pmatrix} \mathbb{1}_{16} & 0 \\ 0 & -\mathbb{1}_{16} \end{pmatrix} , \tag{A.3}$$

The multi-indexed gamma matrices are defined to be,

$$\gamma^{\mu_1 \cdots \mu_{2n}\alpha}{}_\beta = (\gamma^{[\mu_1} \bar{\gamma}^{\mu_2} \cdots \bar{\gamma}^{\mu_{2n}]})^\alpha{}_\beta , \qquad \bar{\gamma}^{\mu_1 \cdots \mu_{2n}}{}_\alpha{}^\beta = (\bar{\gamma}^{[\mu_1} \gamma^{\mu_2} \cdots \gamma^{\mu_{2n}]})_\alpha{}^\beta ,$$

$$\gamma^{\mu_1\cdots\mu_{2n+1}\alpha\beta} = (\gamma^{[\mu_1}\bar\gamma^{\mu_2}\cdots\gamma^{\mu_{2n+1}]})^{\alpha\beta} \;, \qquad \bar\gamma_{\alpha\beta}^{\mu_1\cdots\mu_{2n+1}} = (\bar\gamma^{[\mu_1}\gamma^{\mu_2}\cdots\bar\gamma^{\mu_{2n+1}]})_{\alpha\beta} \;, \tag{A.4}$$

The Poincaré duality properties are given by,

$$\begin{aligned}
\gamma^{\mu_1\cdots\mu_n} &= -\frac{1}{(10-n)!}\epsilon^{\mu_1\cdots\mu_{10}}\gamma_{\mu_{n+1}\cdots\mu_{10}}\begin{cases}(-1)^{\frac{n}{2}}\,, & n=\text{even}\,,\\ (-1)^{\frac{n+1}{2}}\,, & n=\text{odd}\,,\end{cases}\\
\bar\gamma^{\mu_1\cdots\mu_n} &= \frac{1}{(10-n)!}\epsilon^{\mu_1\cdots\mu_{10}}\bar\gamma_{\mu_{n+1}\cdots\mu_{10}}\begin{cases}(-1)^{\frac{n}{2}}\,, & n=\text{even}\,,\\ (-1)^{\frac{n+1}{2}}\,, & n=\text{odd}\,,\end{cases}
\end{aligned} \tag{A.5}$$

Various trace formulas are,

$$\begin{aligned}
\mathrm{Tr}(\gamma^{\mu_1\cdots\mu_n}) &= 0\,,\\
\mathrm{Tr}(\gamma^{\mu_1\cdots\mu_{2n}}\gamma^{\nu_1\cdots\nu_m}) &= 0\,, \quad m\neq 2n\,,\\
\mathrm{Tr}(\gamma^{\mu_1\cdots\mu_{2n+1}}\bar\gamma^{\nu_1\cdots\nu_m}) &= 0\,, \quad m\neq 2n+1\,,\\
\mathrm{Tr}(\gamma^{\mu_1\cdots\mu_{2n}}\gamma^{\nu_1\cdots\nu_{2n}}) &= (-1)^n 16\Delta^{[\mu_1\cdots\mu_{2n}],[\nu_1\cdots\nu_{2n}]}\,, \quad n=1,2\,,\\
\mathrm{Tr}(\gamma^{\mu_1\cdots\mu_{2n+1}}\bar\gamma^{\nu_1\cdots\nu_{2n+1}}) &= (-1)^n 16\Delta^{[\mu_1\cdots\mu_{2n+1}],[\nu_1\cdots\nu_{2n+1}]} + 16\delta_{n,2}\epsilon^{\mu_1\cdots\mu_5\nu_1\cdots\nu_5}\,, \; n=0,1,2\,,
\end{aligned} \tag{A.6}$$

where,

$$\begin{aligned}
\Delta^{[\mu_1\cdots\mu_n],[\nu_1\cdots\nu_n]} &\equiv \sum_{\mathcal{P}}\mathrm{sign}\mathcal{P}\;\eta^{\{\mu_1\cdots\mu_n\},\mathcal{P}\{\nu_1\cdots\nu_n\}}\,,\\
\eta^{\{\mu_1\cdots\mu_n\},\{\nu_1\cdots\nu_n\}} &\equiv \eta^{\mu_1\nu_1}\cdots\eta^{\mu_n\nu_n}\,.
\end{aligned} \tag{A.7}$$

The sum in the first line goes over $n!$ terms. $\{\mu_1\cdots\mu_n\}$ is an ordered set and $\mathcal{P}\{\mu_1\cdots\mu_n\}$ is another ordered set obtained by applying the permutation $\mathcal{P}$ on $\{\mu_1\cdots\mu_n\}$. Relations obtained by interchanging γ and $\bar\gamma$ matrices in eqs. (A.6) also hold. Using these above traces one can prove the following Fiertz identities,

$$\begin{aligned}
\chi^\alpha\xi^\beta &= \frac{1}{16}(\chi\bar\gamma^\mu\xi)\gamma_\mu^{\alpha\beta} + \frac{1}{16\times 3!}(\chi\bar\gamma^{\mu_1\cdots\mu_3}\xi)\gamma^{\alpha\beta}_{\mu_1\cdots\mu_3} + \frac{1}{16\times 5!\times 2}(\chi\bar\gamma^{\mu_1\cdots\mu_5}\xi)\gamma^{\alpha\beta}_{\mu_1\cdots\mu_5}\,,\\
\chi_\alpha\xi_\beta &= \frac{1}{16}(\chi\gamma_\mu\xi)\bar\gamma^\mu_{\alpha\beta} + \frac{1}{16\times 3!}(\chi\gamma_{\mu_1\cdots\mu_3}\xi)\bar\gamma^{\mu_1\cdots\mu_3}_{\alpha\beta} + \frac{1}{16\times 5!\times 2}(\chi\gamma_{\mu_1\cdots\mu_5}\xi)\bar\gamma^{\mu_1\cdots\mu_5}_{\alpha\beta}\,,\\
\chi^\alpha\xi_\beta &= \frac{1}{16}(\chi\xi)\delta^\alpha{}_\beta + \frac{1}{16\times 2!}(\chi\bar\gamma^{\mu_1\mu_2}\xi)\gamma_{\mu_1\mu_2}{}^\alpha{}_\beta + \frac{1}{16\times 4!}(\chi\bar\gamma^{\mu_1\cdots\mu_4}\xi)\gamma_{\mu_1\cdots\mu_4}{}^\alpha{}_\beta\,,\\
\chi_\alpha\xi^\beta &= \frac{1}{16}(\chi\xi)\delta_\alpha{}^\beta + \frac{1}{16\times 2!}(\chi\gamma^{\mu_1\mu_2}\xi)\bar\gamma_{\mu_1\mu_2\alpha}{}^\beta + \frac{1}{16\times 4!}(\chi\gamma^{\mu_1\cdots\mu_4}\xi)\bar\gamma_{\mu_1\cdots\mu_4\alpha}{}^\beta\,.
\end{aligned} \tag{A.8}$$

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